Some NP-Complete Knots

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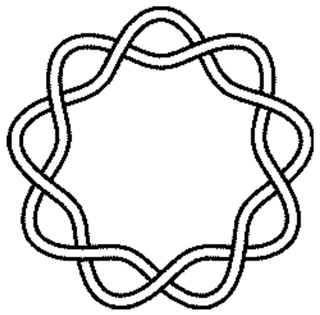
Abstract

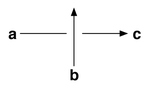
This work demonstrates the computational classification of over one hundred knots using a single quandle. The notions of NP-complete quandles and knots will be introduced along with quandle colorings of each classified knot. The knots in question include those of Rolfsen's Knot table along with many torus knots.

9_1 Knot

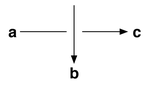


Knot Coloring

Definition 1. A coloring of a knot $\mathcal{K}$ by a quandle Q is a labeling of the arcs of a knot $\mathcal{K}$ by the elements of Q that uses at least two elements of Q and satisfies the following conditions at each crossing.



$a \triangleright b = c$



$a = c \triangleright b$

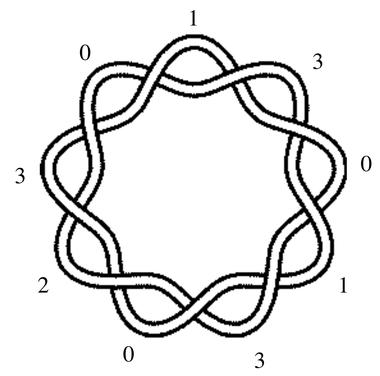
Wood6

$\triangleright$	0	1	2	3	4	5
0	0	3	1	4	2	0
1	2	1	5	0	1	3
2	4	0	2	2	5	1
3	1	5	3	3	0	4
4	3	4	0	5	4	2
5	5	2	4	1	3	5

- Wood6 is NP-Complete.
- Any coloring by Wood6 is a full coloring.

9_1 Colored

```
[0,3,1,0,3,1,0,3,1]
```



- 9_1 is the torus knot $T(2, 9)$.
- $T(2, q)$ is Wood6 colorable for q divisible by 3 but not by 2.

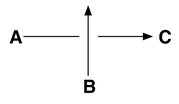
Wood6, Prolog-Style

Here is the definition of q/3 corresponding to Wood6.

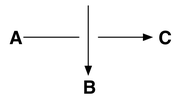
```
q(0,1,3). q(0,2,1). q(0,3,4). q(0,4,2). q(0,5,0).
q(1,0,2). q(1,2,5). q(1,3,0). q(1,4,1). q(1,5,3).
q(2,0,4). q(2,1,0). q(2,3,2). q(2,4,5). q(2,5,1).
q(3,0,1). q(3,1,5). q(3,2,3). q(3,4,0). q(3,5,4).
q(4,0,3). q(4,1,4). q(4,2,0). q(4,3,5). q(4,5,2).
q(5,0,5). q(5,1,2). q(5,2,4). q(5,3,1). q(5,4,3).

q(X,X,X).
```

Crossings in Prolog



$q(A, B, C)$.



$q(C, B, A)$.

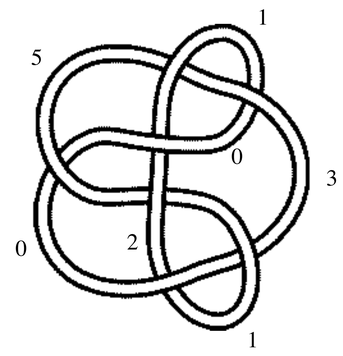
Coloring in Prolog

This is a Prolog predicate that finds all colorings of 7_7 by the quandle defined by q/3.

```
color7_7([X1,X2,X3,X4,X5,X6,X7]) :-
   q(X7,X5,X1),
   q(X2,X6,X1),
   q(X2,X1,X3),
   q(X4,X7,X3),
   q(X4,X3,X5),
   q(X6,X2,X5),
   q(X7,X4,X6).
```

7_7

```
[0,0,1,2,1,5,3]
```



Full Colorings

- A full coloring uses a generating set for Q.
- If Q is NP-Complete then a full coloring shows $\mathcal{K}$ is NP-Complete.

Some NP-Complete Knots

Crossings	Rank
3	1
6	1
7	4, 7
8	5, 10, 11, 15, 18, 19, 20, 21
9	1, 2, 4, 6, 10, 11, 15, 16, 17, 23, 24, 28, 29, 34, 35, 37, 38, 40, 46, 47, 48
10	5, 9, 10, 14, 19, 21, 29, 31, 32, 36, 40, 42, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 82, 84, 85, 87, 89, 96, 97, 98, 99, 103, 106, 107, 108, 112, 113, 114, 120, 122, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 158, 159, 160, 163, 164, 165

Future Work

- Torus Knots
- Two-Bridge Knots
- More NP-Complete connected quandles
- Tractable Knots?

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